Algebraic and p-adic fundamental groups

Organizers: Ngo Dac Tuan (CNRS and University Claude Bernard Lyon 1, France)
Jérôme Poineau (University of Caen Normandy, France)

Date: September 12–13, 2019
Place: Caen, France

Thursday September 12

09:30 – 10:30 **Christopher Lazda** (Univ. Warwick)

A homotopy exact sequence for overconvergent fundamental groups

The overconvergent fundamental group of a variety in characteristic p is that attached to the category of overconvergent isocrystals via Tannakian duality. If $f : X \to S$ is a smooth, projective morphism of smooth varieties, with fibre X_s, I will explain how to show that the associated sequence of overconvergent fundamental groups $\pi_1(X_s) \to \pi_1(X) \to \pi_1(S) \to 1$ is exact. I will also discuss a couple of applications, including a Lefschetz style theorem as well as a comparison result between convergent and étale fundamental groups. This is joint work with Ambrus Pál.

10:30 – 11:00 **Coffee break**

11:00 – 12:00 **Marco D'Addezio** (MPIM Bonn)

Slope filtration and monodromy groups of F-isocrystals

Let X be a smooth variety over a finite field. By the theory of weights, the neutral component of the monodromy group of a "motivic" overconvergent isocrystal over X is semi-simple. On the other hand, the subgroup induced by the convergent realization is quite mysterious. In the explicit cases computed by Crew, it is the parabolic subgroup induced by the slope filtration. I will talk about a work in progress which aims to shed some light on this subgroup. I will also explain an application of the result obtained so far to separable points of abelian varieties.

12:00 – 14:00 **Lunch**

14:00 – 15:00 **Katharina Hübner** (Univ. Heidelberg)

The tame site

For a scheme of characteristic $p > 0$ (or mixed characteristic) étale cohomology with p-torsion coefficients does not behave very well: Smooth base change, cohomological purity, homotopy invariance, just to name a few, only hold for coefficients prime to the characteristic. The reason for this failure is the existence of wild ramification. For issues concerning fundamental groups this has been addressed by introducing the tame fundamental group. This talk presents a modification of the étale topology that does not admit for wild ramification, called the tame site. Its fundamental group is the tame fundamental group that has already been studied. For coefficients away from the characteristic the étale and tame cohomology groups are isomorphic and for p-torsion coefficients they are better behaved than the étale cohomology groups.

15:15 – 16:15 **Emmanuel Lepage** (Jussieu)

Absolute anabelian conjecture for p-adic curves with resolution of non-singularities

The anabelian conjecture for curves over p-adic fields asks if an isomorphism of étale fundamental groups of hyperbolic curves over p-adic fields comes from an isomorphism of curves. A "relative" version, where the isomorphism is assumed to lie over a geometric isomorphism of Galois groups of the base p-adic fields, was proven by Mochizuki. But the "absolute" version, without any assumption on the Galois groups, was only known for a very special kind of curves, called Belyi curves. The goal of the lecture is to show the absolute anabelian conjecture, for a wider family of curves, including for example Mumford curves.

16:30 – 17:00   **Coffee break**

17:00 – 18:00   **Colloquium talk by Hélène Esnault** (FU Berlin)

> *Vanishing theorems in algebraic-arithmetic geometry*

Classically they are of two sorts: Andreotti-Frankel for Stein manifolds, analogously Artin vanishing on affine varieties, and Artin-Schreier theory on proper varieties in positive characteristic. We will explain Scholze's vanishing theorem which holds in characteristic zero and mixes the two theories, and mention our proof of it, which holds in positive characteristic (and thus answers Scholze's conjecture).

Friday September 13

09:15 – 10:15   **Hélène Esnault** (FU Berlin)

> *Étale cohomology of rank one ℓ-adic local systems in positive characteristic*

We show that in positive characteristic special loci of deformation spaces of rank one ℓ-adic local systems are quasi-linear. From this we deduce the Hard Lefschetz theorem for rank one ℓ-adic local systems and a generic vanishing theorem. (Joint with Moritz Kerz)

10:15 – 10:45   **Coffee break**

10:45 – 11:45   **Fabio Tonini** (Univ. Florence)

> *Representations of the Nori fundamental group and variants*

The Nori fundamental group scheme of a scheme X is a profinite group scheme that "controls" torsors over X under finite group schemes. In the talk I will describe its category of representations by considering vector bundles with extra structure. I will also talk about a variant of this group, the local fundamental group scheme.

11:45 – 13:30   **Lunch**

13:30 – 14:30   **Ho Hai Phùng** (IMH Hanoi)

> *On the relative differential fundamental group schemes of a proper smooth scheme over a discrete valuation ring*

Let X be a smooth scheme over a discrete valuation ring R. The relative differential fundamental group scheme of X/R is defined from the category of stratified bundles on X/R by means of Tannakian duality. The aim of our work is to give a coarse construction of this (affine) group scheme and some properties. This is a joint work with N.D. Duong (Hanoi) and J.P.P dos Santos (Jussieu).

14:45 – 15:45   **João Pedro Dos Santos** (Jussieu)

> *Finite torsors on schemes defined over a DVR*

In this talk I shall report on a theory proposed by myself and P. H. Hai defining an analogue of Nori's theory of the fundamental group scheme in the case of projective schemes over a discrete valuation ring A.

I shall begin by briefly explaining Nori's theory of the fundamental group scheme through three approaches (semi-stability, "filtering" and "trivializing") and then introduce the analogous questions for schemes over A.

After talking about the filtering version proposed by Gasbarri (and reviewed by Antei, Emsalem and Gasbarri), I will explain how the "trivializing" alternative allows to identify certain *Tannakian* categories of coherent modules on a projective A-scheme. Following this, I comment on how to put all the aforementioned categories together to form a single Tannakian category and a single flat group scheme Π^{tr}.

In the rest of the talk I will put forward the salient properties of Π^{tr} as *strict pro-finiteness* (which amounts to showing that quasi-finite group schemes play no role), fibre-by-fibre characterisation and its relation to the other approaches.

List of participants

(1) Marco D'Addezio (MPIM Bonn)
(2) Bruno Anglès (Caen)
(3) Tiphaine Beaumont (Caen)
(4) Angelot Behajaina (Caen)
(5) Dorian Berger (Caen)
(6) Velibor Bojković (Caen)
(7) Vincent Bosser (Caen)
(8) John Boxall (Caen)
(9) Daniel Caro (Caen)
(10) Rodrigo Codorniu (Nice)
(11) Hélène Esnault (FU Berlin)
(12) Sylvain Gaulhiac (Jussieu)
(13) Katharina Hübner (Heidelberg)
(14) Theresa Kumpitsch (Frankfurt)
(15) Amine Laaroussi (Lille)
(16) Marcin Lara (IMPAN Warsaw)
(17) Christopher Lazda (Warwick)
(18) Huy Hung Le (Caen)
(19) Emmanuel Lepage (Jussieu)
(20) Vlerë Mehmeti (Caen)
(21) Ruben Muñoz-Bertrand (Caen)
(22) Tuan Ngo Dac (CNRS / Lyon 1)
(23) Quang Do Thong Nguyen (Besançon)
(24) Ho Hai Phung (IMH Hanoi / Caen)
(25) Jérôme Poineau (Caen)
(26) João Pedro Dos Santos (Jussieu)
(27) Philippe Satgé (Caen)
(28) Floric Tavares Ribeiro (Caen)
(29) Fabio Tonini (Florence)